

Indian Statistical Institute, Bangalore

M. Math. First Year

Second Semester - Algebra II

Mid-Semester Exam

Duration: 3 hours

Date : February 23, 2016

Max Marks: 100

Note: Answer all questions. Your answers should be clear, complete and to the point.

1. In S_5 , find

- (a) precisely one representative from each conjugacy class of elements;
- (b) the number of conjugacy classes of elements of order 5; and
- (c) a subgroup of order 8.

Justify your answer.

[8+8+9]

2. Define a nilpotent group. Show that a nilpotent group of order n has a subgroup of order m for each divisor m of n . [3+12]

3. Determine the number of non-abelian groups of order 55, upto isomorphism. [15]

4. Define precisely a free group on a set X and presentation of a group. Write a presentation for a free abelian group on n elements. [4+4+7]

5. Find the order of the group $GL(2, \mathbb{F}_2)$. Is it simple? [15]

6. (a) Give an example of an irreducible polynomial of degree 4 on \mathbb{F}_2 ;

- (b) Let $q = p^r, p$ a prime. Determine the integers t, k and $a \in \mathbb{F}_q$ for which the evaluation of the polynomial $(X^{p^t} - a)^k$ gives a bijective map from \mathbb{F}_q to itself.

[8+7]